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A SIMPLIFIED STUDY OF THE TURBULENT FREE SHEAR LAYER BETWEEN TWO DISSIMILAR GASES

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LIST OF SYMBOLS *

a	spreading rate parameter = δ/x
c_p	specific heat at constant pressure
c_α	local mass fraction of the region 1 gas
c_β	local mass fraction of the region 2 gas
$\dot{e}$	total energy transfer rate per unit area
h	local enthalpy
ℓ	mixing length (see Eq. 3.22)
M	local Mach number
m	local molecular weight
n	local particle density
p	local pressure
$\dot{q}$	local heat transfer rate per unit area
R	local gas constant
$\mathcal{R}$	universal gas constant
St	Stanton number = $\dot{e}/\rho_1 u_1 (h_1^0 - h_2)$
T	local temperature
t	time
u	local velocity in the x direction
v	local velocity in the y direction
x	coordinate parallel to the initial region 1 flow
y	coordinate normal to x
γ	ratio of specific heats
δ	total thickness of the mixing region (at a given x)
η	radial coordinate = y/δ
κ	shear stress proportionality constant (see Eq. 3.23)
ρ	local density
τ	turbulent shearing stress (see Eq. 3.22)

* Additional symbols introduced in the appendix are defined in the appendix.

Subscripts

0	conditions along the coordinate $\eta = 0$
1	conditions in region 1
2	conditions in region 2
*	conditions along the dividing streamline (η_*)
α	due to gas from region 1
β	due to gas from region 2

Superscripts

0	stagnation condition

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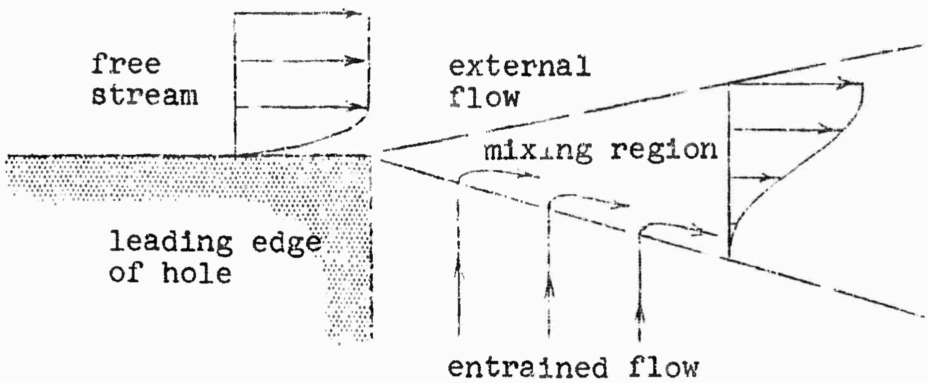
1. INTRODUCTION

An analytical study of the turbulent mixing of two dissimilar gases in a free shear layer was performed by Aeronautical Research Associates of Princeton, Inc. for the Hypervelocity Kill Mechanisms Program. It was felt desirable to develop a simplified method by which closed form expressions could be obtained for the characteristics of such a flow. These included the boundaries of the mixing region, the location of the dividing streamline, the local flow velocities, the local thermodynamic conditions, and the mean rate of energy transport across the shear layer.

It will be shown that although the effects of compressibility and the presence of dissimilar gases on either side of the mixing region raise the level of complexity of the problem, the approach remains the same as that utilized to solve the single gas incompressible problem. Therefore, in order to focus attention on this approach, the incompressible one-gas flow is treated first (Section 3), then the modifications required to account for compressibility are shown and the results, which then become functions of Mach number and enthalpy are given (Section 4), and finally, the treatment and results for the mixing of two dissimilar gases are presented (Section 5).

2. THE FREE SHEAR LAYER

Before proceeding with the solution of the incompressible single gas, the compressible single gas, and finally the compressible two-gas problems, a more detailed description of the flow pattern under study and the corresponding model utilized throughout the analysis should be given. Shown below is a sketch of a cross-section of a free shear layer showing the oncoming free stream, the region of turbulent mixing, and the entrained flow. Typical velocity profiles of the free stream and mixing layer are also shown.

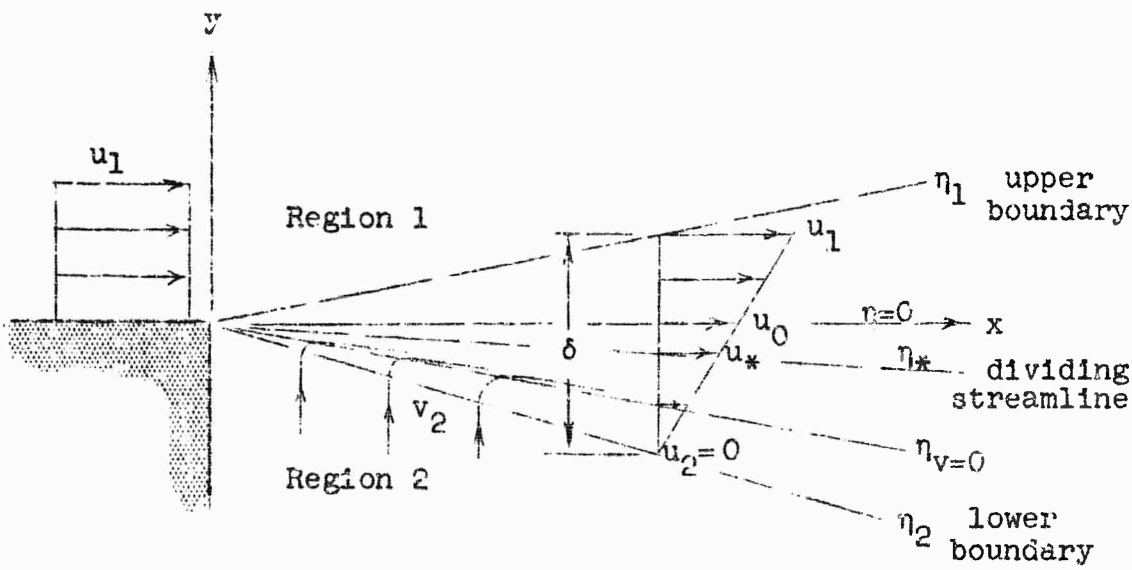


Sketch of Actual Shear Layer

Several modifications to the above picture can be introduced which, while simplifying the analysis, do not, in general, materially effect the solution. For one, the boundary layer buildup in the free stream prior to its arrival at the leading edge of the hole is assumed to have little effect upon the subsequent mixing and thus is neglected. For another, the exact self-similar horizontal velocity profile in the mixing region can be approximated by a properly chosen straight line. It is recognized that analytical expressions for the actual profile do exist (for example, reference 2); however, a linear profile, particularly for the two-gas compressible flow problem, allows the development of the solution in terms of closed form expressions which are most instructive and, for our purposes, appear adequately accurate. The method by which the proper equivalent profile was obtained is discussed in the next section.

Several assumptions common in boundary layer theory were also made. Specifically they were that the gradients in the vertical direction are much greater than those in the horizontal direction, the static pressure is everywhere a constant, and the mean flow is steady.

The use of these assumptions leads to the idealized model for a shear layer shown in the next sketch. A number of important quantities which will be discussed in the development to be presented in the next several sections have been indicated.



Sketch of Model for Shear Layer Studies

3. INCOMPRESSIBLE SINGLE-GAS FLOW

In this initial analysis, several flow parameters of major interest will be derived for the incompressible single-gas shear layer. For this case the method will be most transparent and experimental data are available for comparison. Having established the pattern of the development with this simple case, it will not be necessary to retrace the steps in detail for each subsequent case. In the ensuing sections, the emphasis will be upon the modifications required for treatment of the more general flows and the subsequent effects upon the various flow parameters of interest.

3.1. Horizontal velocity.

If one assumes, as is customary in turbulent shear layer theory, that the mean profiles are self-similar and that mixing length is proportional to shear layer breadth, it can be shown that the total spread of the layer varies linearly with downstream distance and that the velocity and hence all other flow parameters remain constant along any ray emanating from the origin. Thus the problem can be expressed in terms of only one independent variable η where $\eta = y/x$.

Expressing the statements that the horizontal velocity is linear in y and that the shear layer thickness is linear in x as

$$u = u_o + \left(\frac{u_1}{\delta}\right)y \tag{3.1}$$

and

$$\delta = ax \tag{3.2}$$

we can write in terms of the single coordinate $\eta = y/\delta$

$$\frac{u}{u_1} = \frac{u_0}{u_1} + \frac{\eta}{a} \tag{3.3}$$

from which it can be shown that

$$\eta_1/a = 1 - \frac{u_0}{u_1} \quad \text{and} \quad \eta_2/a = -\frac{u_0}{u_1} \tag{3.4}$$

where η_1 , η_2 , u_1 , u_0 , and δ are shown in the final sketch of the last section.

The reference velocity u_0 in turn can be obtained by utilizing the fact that the total horizontal momentum flux in the mixing region may be written

$$\int_{y_2}^{y_1} \rho u^2 dy = \rho_1 u_1^2 y_1 \tag{3.5a}$$

where y_1 and y_2 are the limits of the mixing region at x . Transposing to the coordinate η and nondimensionalizing, this may be written

$$\int_{\eta_2/a}^{\eta_1/a} \left(\frac{u}{u_1}\right)^2 d\left(\frac{\eta}{a}\right) = \frac{\eta_1}{a} \tag{3.5b}$$

Introducing the relationships (3.3) and (3.4) for u/u_1 and the limits η_1/a and η_2/a and performing the indicated integration, we find

$$\frac{u_0}{u_1} = \frac{2}{3} \tag{3.6}$$

Finally, returning to the above equations for the horizontal velocity and mixing region limits, we can now write

$$\frac{u}{u_1} = \frac{2}{3} + \frac{\eta}{a} \tag{3.7}$$

$$\frac{\eta_1}{a} = \frac{1}{3} \qquad (3.8)$$

$$\frac{\eta_2}{a} = -\frac{2}{3} \qquad (3.9)$$

3.2. Vertical velocity.

The vertical velocity anywhere in the flow can be obtained through the use of the continuity equation. For incompressible flow, this equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3.10)$$

Using (3.7) and replacing η by y/x, we can write

$$\frac{\partial u}{\partial x} = -\frac{u_1}{a}\frac{y}{x^2} \qquad (3.11)$$

or by the use of (3.10)

$$\frac{\partial v}{\partial y} = +\frac{u_1}{a}\frac{y}{x^2} \qquad (3.12)$$

Integrating this last expression yields

$$\frac{v}{u_1} = \frac{\eta^2}{2a} + C \qquad (3.13a)$$

where $C = -\frac{\eta_1^2}{2a}$ from the boundary condition $v = 0$ at $\eta = \eta_1$. Thus

$$\frac{v}{u_1} = \frac{a}{2}\left[\left(\frac{\eta}{a}\right)^2 - \left(\frac{\eta_1}{a}\right)^2\right] \qquad (3.13b)$$

or by substituting (3.8)

$$\frac{v}{u_1} = \frac{a}{2}\left[\left(\frac{\eta}{a}\right)^2 - \frac{1}{9}\right] \tag{3.13c}$$

A particular vertical velocity that may be of interest is the one that exists at the lower edge of the mixing region, i.e. the velocity at which the gas initially at rest is being entrained. By substituting the expression η_2/a from (3.9) for η/a in the above equation, we obtain

$$\frac{v_2}{u_1} = \frac{a}{6} \tag{3.14}$$

3.3. Dividing streamline.

Of particular interest in several future computations will be conditions on the dividing streamline. The dividing streamline of the mixing region is defined as that streamline above which the mass flow is equal to the mass flow from the external stream that has been captured by the mixing region. Thus

$$\int_{y_*}^{y_1} \rho u dy = \rho_1 u_1 y_1 \tag{3.15a}$$

where the subscript $*$ will be used to denote conditions on the dividing streamline. In terms of the variable η,

$$\int_{\eta_*/a}^{\eta_1/a} \left(\frac{u}{u_1}\right) d\left(\frac{\eta}{a}\right) = \frac{\eta_1}{a} \tag{3.15b}$$

Again utilizing (3.7) and (3.8) for u/u_1 and η_1/a, a quadratic expression for η_*/a is obtained from which one deduces that

$$\frac{\eta_*}{a} = -\frac{2}{3} + \sqrt{\frac{1}{3}} = -.0893 \qquad (3.16)$$

(Note that the plus sign of the square root is to be taken since η_*/a must lie inside $\eta_2/a = -2/3$.)

3.4. Deflection of the free stream.

In the derivations for the vertical velocity and the dividing streamline, it was assumed that the free stream velocity contained no vertical component $(v_1 = 0)$. If, instead, it is assumed that due perhaps to the presence of a shock emanating from the upstream edge of the hole there is a vertical velocity component to the free stream, then the general vertical velocity and dividing streamline expressions are modified as follows:

The constant of integration C in equation (3.13a) is now $C = (v_1/u_1) - (\eta_1^2/2a)$ so that

$$\frac{v}{u_1} = \frac{a}{2}\left[\left(\frac{\eta}{a}\right)^2 - \frac{1}{9}\right] + \frac{v_1}{u_1} \qquad (3.17)$$

and

$$\frac{v_2}{u_1} = \frac{a}{6} + \frac{v_1}{u_1} \qquad (3.18)$$

The mass balance equation (3.15a) must contain a term to account for the mass flux being lost through the upper boundary of the mixing region. Thus, instead of (3.15a) we have

$$\int_{y_*}^{y_1} \rho u dy = \rho_1 u_1 y_1 - \rho_1 v_1 x \tag{3.19}$$

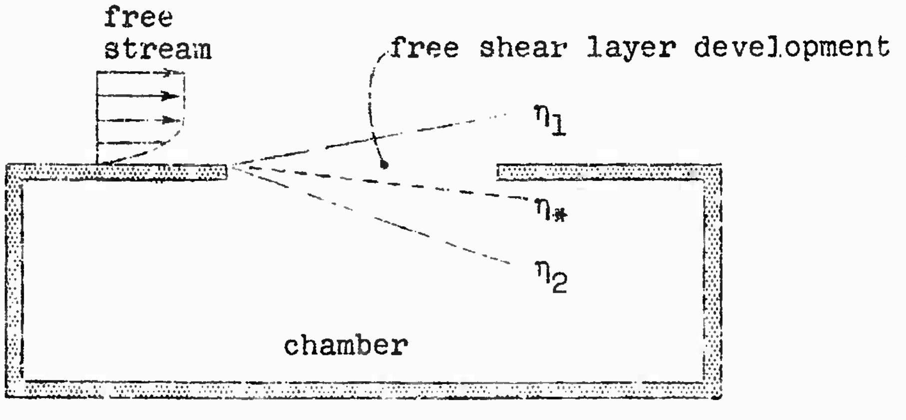
Nondimensionalizing and integrating as before yields

$$\frac{\eta_*}{a} = -\frac{2}{3} + \sqrt{\frac{1}{3} + \frac{2}{a}\frac{v_1}{u_1}} \tag{3.20}$$

An interesting quantity which can now be computed is the magnitude of the vertical velocity component of the free stream necessary to drive the dividing streamline to a horizontal position, i.e. $\eta_* = 0$. It is quickly seen from the above expression that

$$\left(\frac{v_1}{u_1}\right)_{\eta_*=0} = \frac{a}{18} \tag{3.21}$$

If one assumes that the shear layer is formed as a result of a uniform flow across an orifice in an otherwise closed thin-walled chamber, then as long as the dividing streamline is deflected downward the mass in the charber will increase (see sketch below).



In this case, to the order of accuracy of the present shear layer model, the resulting increased pressure in the chamber should deflect the free stream upward until the pressure is just such that a shock is formed which deflects the exterior flow by an amount v_1 as given by (3.21). At this point no further increase in mass and thus pressure in the cavity is obtained since $\eta_* = 0$ and thus the flow is stabilized. For finite thickness walls, the deflection of η_* necessary to arrest the fill-up and the associated v_1 can be similarly computed.

3.5. Mixing region growth.

Up to this point, the boundaries of the mixing region and the dividing streamline have been obtained as a percentage of the nondimensional spreading rate $a = \delta/x$. To determine this quantity and thus the magnitudes of η_1, η_2, and η_*, it is necessary to introduce the turbulent stress present in the shear layer. The formulation used for this study is one which assumes that the turbulent stress is given by the Prandtl mixing length formula, namely

$$\tau = \rho \ell^2 \frac{\partial u}{\partial y} \left| \frac{\partial u}{\partial y} \right| \qquad (3.22)$$

where ℓ is the mixing length. Assuming that ℓ is constant across the layer at any station and is proportional to the breadth of the shear layer δ, we can write by making use of our linear velocity profile $\left(\frac{\partial u}{\partial y} = \frac{u_1}{\delta}\right)$

$$\tau = \kappa \rho u_1^2 \qquad (3.23)$$

where κ is referred to as the shear stress proportionality constant. For an actual velocity profile (see reference 2), the line of maximum shear is found to coincide with the dividing streamline. For this reason the characteristic shear stress just derived will be assumed to act on the dividing streamline.

To associate a spreading rate with this shear, we first introduce the momentum equation

$$\frac{\partial \rho u^2}{\partial x} + \frac{\partial \rho u v}{\partial y} = \frac{\partial \tau}{\partial y} \tag{3.24}$$

and integrate this expression from the lower boundary of the mixing region y_2 to the dividing streamline y_*. There results

$$\frac{d}{dx}\int_{y_2}^{y_*} \rho u^2 dy + \rho_2 u_2^2 \frac{dy_2}{dx} - \rho_* u_*^2 \frac{dy_*}{dx} + \rho_2 u_2 v_2 - \rho_* u_* v_* = \tau_* - \tau_2 \tag{3.25}$$

Applying the conditions $u_2 = \tau_2 = 0$ and noting that $dy_*/dx = v_*/u_*$, this expression reduces to

$$\frac{d}{dx}\int_{\eta_2/a}^{\eta_*/a} x\left(\frac{u}{u_1}\right)^2 \frac{d\eta}{a} = \frac{\tau_*}{a\rho_1 u_1^2} = \frac{\kappa}{a} \tag{3.26}$$

Finally, carrying out the indicated integrations and differentiations results in

$$\frac{a}{\kappa} = \left\{\frac{2}{3}\left(\frac{\eta_*}{a} + \frac{2}{3}\right) + \frac{4}{9}\left[\left(\frac{\eta_*}{a}\right)^2 - \frac{4}{9}\right] + \frac{1}{3}\left[\left(\frac{\eta_*}{a}\right)^3 + \frac{8}{27}\right]\right\}^{-1} \tag{3.27}$$

where η_*/a is in general given by (3.20). For the case in which there is no free stream deflection $(v_1 = 0)$, we have, from (3.16), $\eta_*/a = -.0893$ and the spreading rate becomes

$$a = 15.6\kappa \tag{3.28}$$

It appears that we have merely substituted one unknown quantity for another. However, shear stress proportionality factors have been experimentally obtained over a wide range of flow conditions both at ARAP and elsewhere. For instance, in reference 1 a shear stress parameter K analogous to κ was obtained from the measurement of free axisymmetric turbulent jets. It was found there that K depends, to first order, only upon a suitably chosen local Mach number. The Mach number used in reference 1 was that found where the local jet velocity had dropped to one-half its value on the centerline for the same axial location, this location being quite close to the point where the stress is a maximum. A plot of K as a function of local Mach number thus defined is given in Figure 1.*

To test the suitability of utilizing the K from reference 1 as a basis for the κ parameter developed here, use was made of calculations of the spreading rate of an incompressible shear layer found in reference 2 and of (3.28) for the incompressible relationship between spreading rate and κ. Reproduced in Figure 2 is the shear layer profile given in the above reference. The proper equivalent linear profile also shown in Figure 2 was constructed such that the momentum lost between y_0 and y_1, i.e. the momentum gained from y_2 to y_0, in the case of the linear profile is equal to the momentum which is lost by the exact

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* The values at K shown in Figure 1 are a factor of 4 less than those in reference 1. This change was necessary to compensate for the difference in the basic definition of τ used here and in the referenced report.

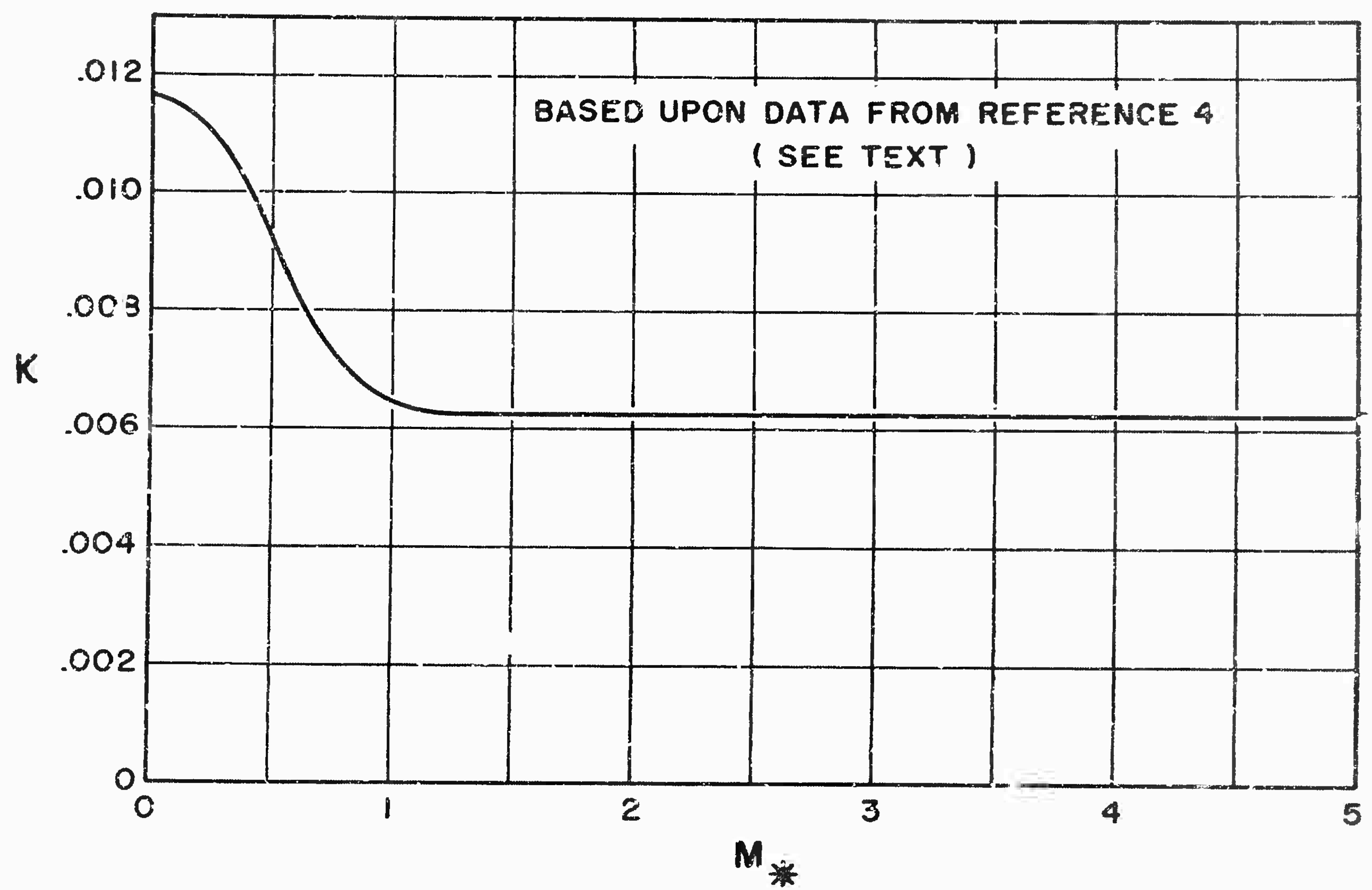

Figure 1. Variation of κ with local Mach number.

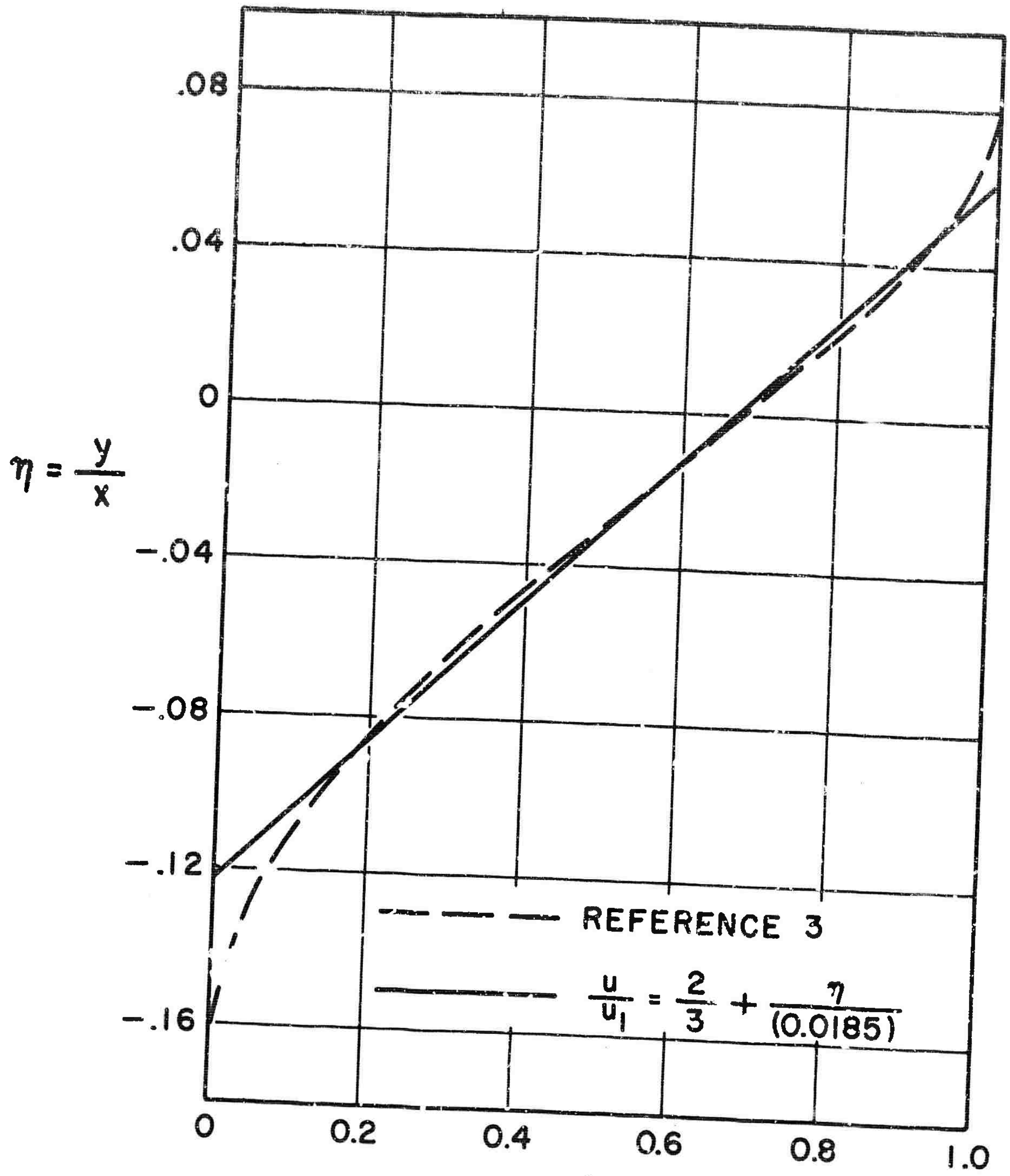

Figure 2. Comparison of Tollmien and linear velocity profiles.

profile between the same two limits. Thus we have required that the momentum transferred be the same for each profile. This condition leads to a value of the spreading rate $a = \delta/x$ of 0.185 for the equivalent linear profile. Returning to (3.28) we find that κ for incompressible flow is then .0119. From Figure 1 it is seen that the incompressible value of κ is .0117 which agrees quite favorably with the above result. Thus we will assume in what follows that the evaluation of κ versus M shown in Figure 1 is valid for free shear layer flow in which M is the local Mach number on the dividing streamline (the location of maximum shear for incompressible free shear layer flows).

3.6. Summary of the incompressible flow results.

Before proceeding to more general flow problems and the determination of the energy transferred to the cavity associated with each of them, it might be useful to summarize the incompressible flow results. Table 3.1 presents the general form of the several expressions previously derived. Table 3.2 provides the specific values of important quantities for the case of $v_1 = 0$ and $\kappa = .0117$.

TABLE 3.1

Incompressible Shear Layer Flow Expressions

$$\frac{u}{u_1} = \frac{2}{3} + \frac{\eta}{a}$$

$$\frac{v}{u_1} = \frac{a}{2}\left[\left(\frac{\eta}{a}\right)^2 - \frac{1}{9}\right] + \frac{v_1}{u_1}$$

$$\frac{\eta_*}{a} = -\frac{2}{3} + \sqrt{\frac{1}{3} + \frac{2}{a}\frac{v_1}{u_1}}$$

$$\frac{a}{\kappa} = \left\{\frac{2}{3}\left(\frac{\eta_*}{a} + \frac{2}{3}\right) + \frac{4}{9}\left[\left(\frac{\eta_*}{a}\right)^2 - \frac{4}{9}\right] + \frac{1}{3}\left[\left(\frac{\eta_*}{a}\right)^3 + \frac{8}{27}\right]\right\}^{-1}$$

TABLE 3.2

Specific Values of Incompressible Flow Parameters for $v_1 = 0$ and $\kappa = .0117$

$u_0/u_1 = .667$ $\qquad$ $u_*/u_1 = .577$

$v_2/u_1 = .0305$ $\qquad$ $v_{\eta_*=0}/u_1 = .0102$

$a = \frac{\delta}{x} = .183$ which corresponds to an angle of $+10.6^\circ$

$\eta_1 = \frac{y_1}{x} = .0610$ which corresponds to an angle of $+3.5^\circ$

$\eta_* = \frac{y_*}{x} = -.0154$ which corresponds to an angle of -0.9°

$\eta_2 = \frac{y_2}{x} = -.122$ which corresponds to an angle of -7.1°

4. COMPRESSIBLE SINGLE-GAS FLOW

When the Mach number in the free stream is appreciable, the density can no longer be considered constant across the layer. In the previous section, the integration of mass flux and momentum flux assumed ρ invariant. Now these integrals must be reevaluated in terms of a varying density.

4.1. Density distribution.

In what follows we will assume, as was done in reference 1 where the validity of the assumption is discussed in some detail, that the mean local enthalpy in the mixing region is related to the mean velocity through the Crocco integral. Thus the local stagnation enthalpy may be written as a linear function of the local velocity.

$$h^0 = Au + B \tag{4.1}$$

Expanding h^0 in terms of a local static enthalpy, this latter quantity can be written as

$$h = Au + B - \frac{1}{2}u^2 \tag{4.2a}$$

and after inserting the boundary condition $h = h_1$ when $u = u_1$ and $h = h_2$ when $u = 0$, h can be expressed as

$$h = h_2 + \left(h_1^0 - h_2\right)\frac{u}{u_1} - \left(h_1^0 - h_1\right)\left(\frac{u}{u_1}\right)^2 \tag{4.2b}$$

Writing now the local density as

$$\rho = \frac{p}{RT} = \frac{c_p p}{Rh} = \frac{\gamma}{\gamma - 1}\frac{p}{h} \tag{4.3}$$

and recalling the assumption that the pressure p is constant across the mixing layer, we obtain

$$\frac{\rho}{\rho_1} = \left[\frac{h}{h_1}\right]^{-1} = \frac{\dfrac{h_1}{h_1^0}}{\dfrac{h_2}{h_1^0} + \left[1 - \dfrac{h_2}{h_1^0}\right]\left(\dfrac{u}{u_1}\right) - \left[1 - \dfrac{h_1}{h_1^0}\right]\left(\dfrac{u}{u_1}\right)^2} \tag{4.4}$$

where h_2/h_1^0 is the stagnation enthalpy ratio across the entire layer and $h_1/h_1^0 = \left[1 + \frac{\gamma_1 - 1}{2} M_1^2\right]^{-1}$ is a function of the free stream Mach number and ratio of specific heats. Equation (4.4) provides the necessary relationship between the density and velocity for use in the above-mentioned integrals. Because of the linear relationship between velocity and location (3.3), these integrals can be obtained in closed form.

4.2. Compressible single-gas flow parameters.

Not all of the flow relationships developed in the last section are modified by compressibility. The derivations of the general expressions for the horizontal and vertical velocities

$$\frac{u}{u_1} = \frac{u_0}{u_1} + \frac{\eta}{a} \tag{3.3}$$

$$\frac{v}{u_1} = \frac{a}{2}\left[\left(\frac{\eta}{a}\right)^2 - \left(\frac{\eta_1}{a}\right)^2\right] \tag{3.13b}$$

and the expression for the mixing region limits

$$\frac{\eta_1}{a} = 1 - \frac{u_0}{u_1} \quad \text{and} \quad \frac{\eta_2}{a} = -\frac{u_0}{u_1} \tag{3.4}$$

did not involve the density and thus are unaltered.

However, the reference velocity u_0/u_1, the spreading rate a/κ, and the dividing streamline η_*/u obtained as a result of momentum or mass flux integration must be reevaluated.

Reference velocity. The velocity u_0/u_1 was obtained by equating the total momentum in the shear layer at any point to the captured momentum of the initial flow (see section 3.1). For the compressible shear layer, the integral of equation (3.5b) becomes

$$\int_{\eta_2/a}^{\eta_1/a} \frac{\rho}{\rho_1} \left(\frac{u}{u_1}\right)^2 d\left(\frac{\eta}{a}\right) \tag{4.5}$$

where the density ration is given by equation (4.4). Integration now results in an explicit but rather involved expression for u_0/u_1. Written functionally

$$\frac{u_0}{u_1} = f\left(\frac{h_2}{h_1^0}, M_1, \gamma_1\right) \tag{4.6}$$

The actual expression is given in the appendix. It has been evaluated for several enthalpy ratios over a range of Mach numbers on ARAP's high speed digital computer. The results can be seen in Figure 3. Using this information, η_1/a and η_2/a are immediately found from (3.4).

Dividing streamline. The dividing streamline was obtained from mass flow considerations (see section 3.3). The integral in (3.15b) when the density is included, yields an equation for η_*/a in terms of the compressible parameters h_2/h_1^0, M_1, and γ_1 and the vertical

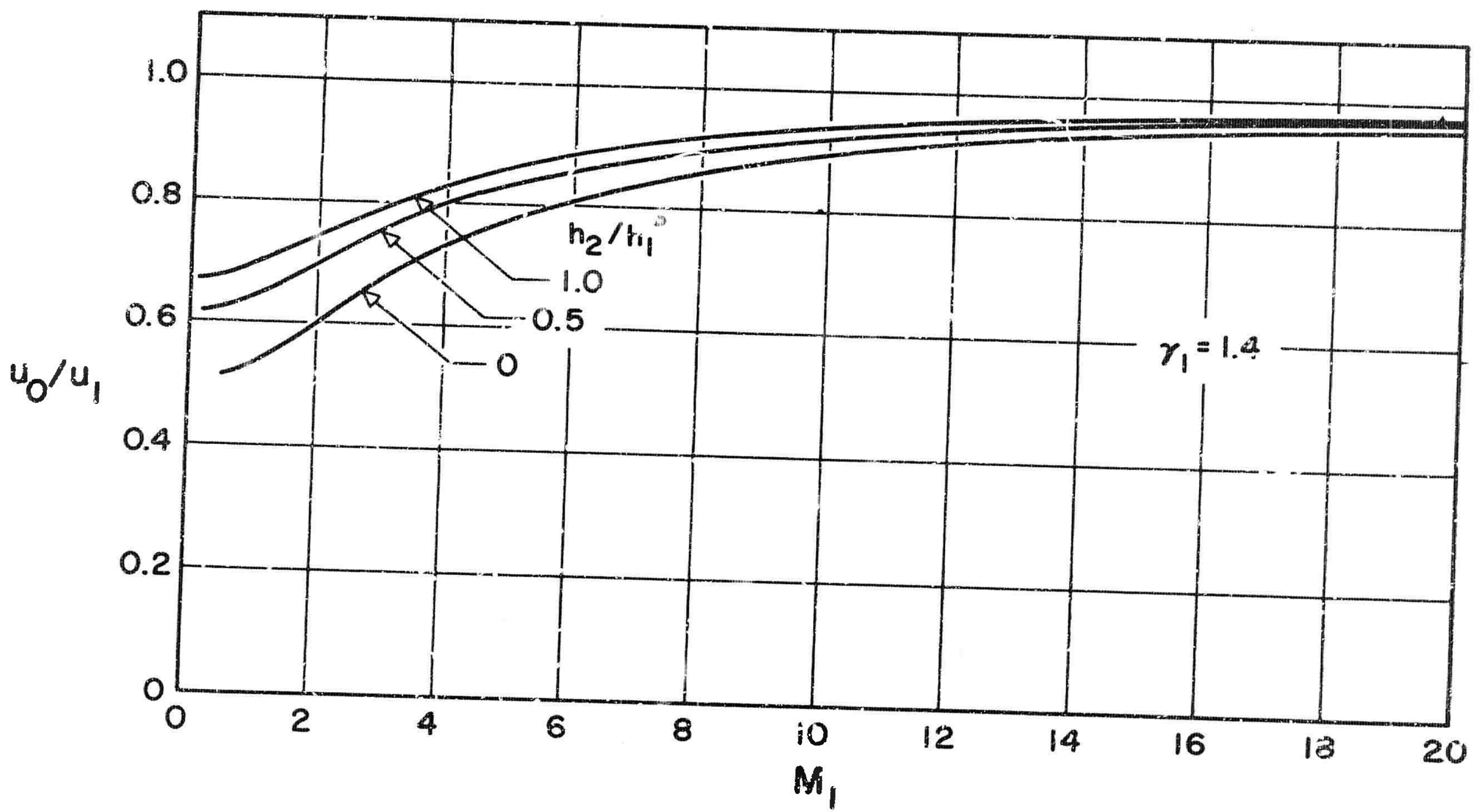

Figure 3. Variation of reference velocity - single gas.

component of the free stream v_1/u_1 . Thus

$$g\left(\frac{\eta_*}{a}, \frac{h_2}{h_1^0}, M_1, \gamma_1, \frac{v_1}{u_1}\right) = 0 \qquad (4.7)$$

(Again, the actual expression is available in the appendix.) However, before evaluation of η_* itself (as well as η_1 and η_2), the spreading rate a must also be recomputed.

<u>Spreading rate.</u> The rate at which the mixing region spreads $a = \delta/x$ was obtained for the incompressible case by computing the ratio of a to the shear stress factor κ . From this ratio and a knowledge of κ , the nondimensional spreading rate δ/x can be obtained. For compressible flows the integration indicated in (3.26) yields

$$\frac{a}{\kappa} = h\left(\frac{h_2}{h_1^0}, M_1, \gamma_1, \frac{\eta_*}{a}\right) \qquad (4.8)$$

(See appendix for actual expression.) The proper value of κ is obtained by first computing M_* as a function of h_2/h_1^0 , M_1 , and γ_1 , and then using the assumed relationship between κ and [illegible] given in Figure 1. The local value of Mach number M_* is shown in Figure 4. The resulting evaluation of $a = \delta/x$ is presented in Figure 5. It is seen that enthalpy ratio across the shear layer has little effect on the total spreading rate.

With the compressible spreading rate known, we can now return to (4.7) and (3.4) and obtain the dividing streamline and mixing region limits, respectively. These quantities are shown as functions of Mach number M_1 and enthalpy ratio h_2/h_1^0 in Figure 6.

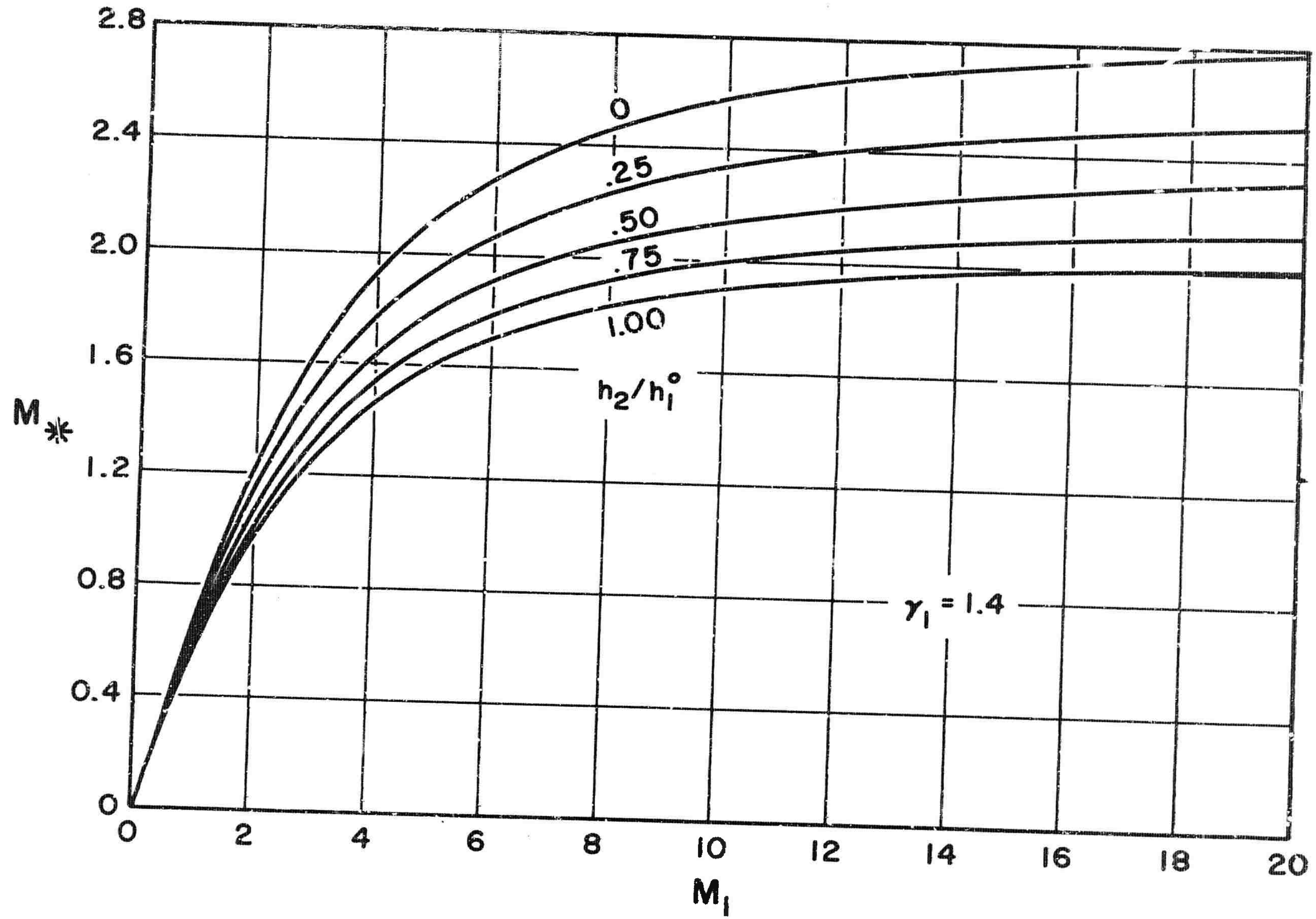

Figure 4. Variation of local Mach number on the dividing streamline - single gas.

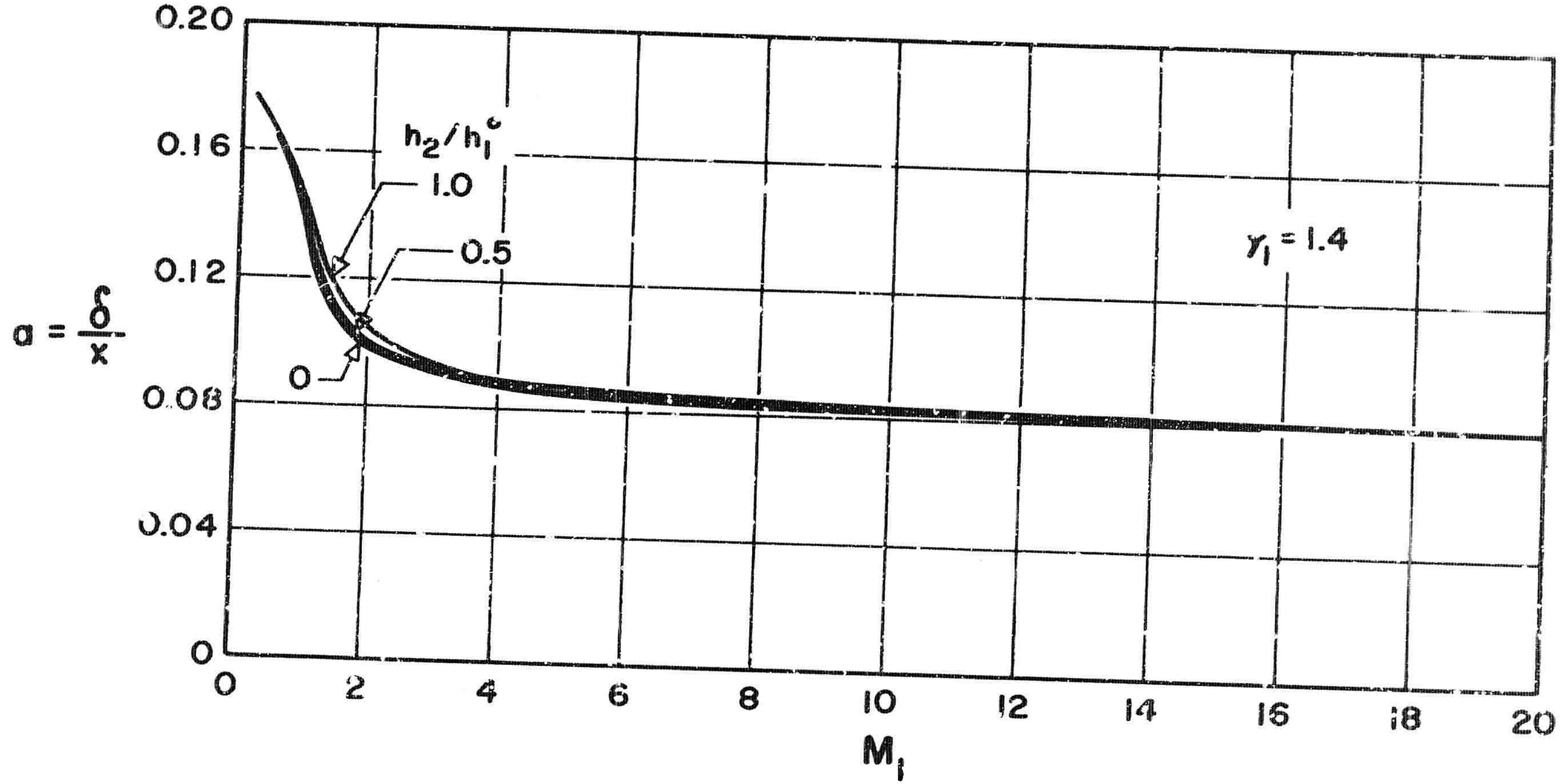

Figure 5. Variation of spreading rate parameter - single gas.

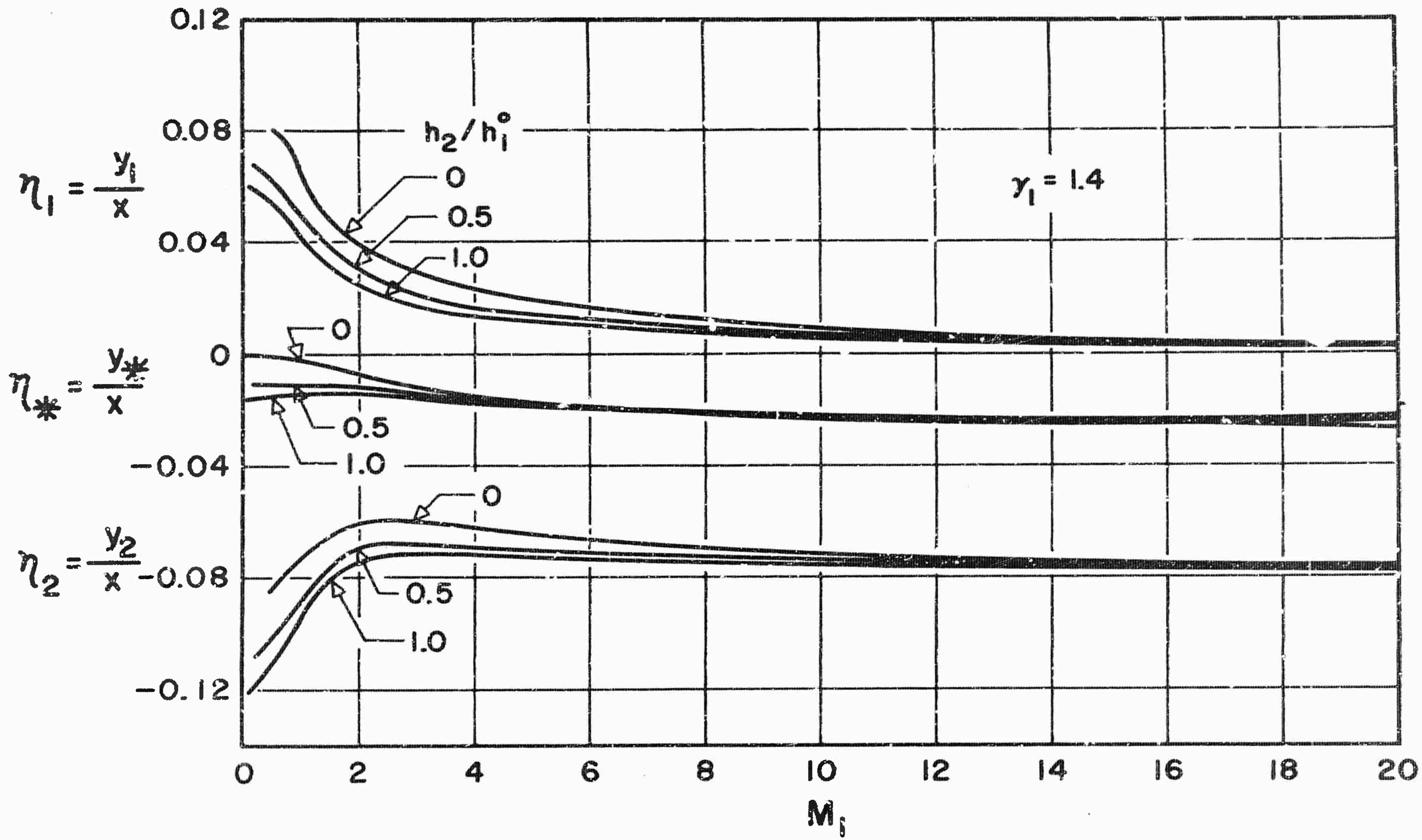

Figure 6. Variation of mixing region boundaries and dividing streamline - single gas.

4.3. Energy transfer.

A characteristic of free shear layer flow that is of interest when there exists a difference between the level of stagnation enthalpy on each side of the layer is the resulting rate at which energy is transferred across the layer. To compute its magnitude, we start by writing that the total energy transferred across a region of the shear layer per unit time per unit area is

$$\dot{e} = \dot{q} + \tau u \tag{4.9}$$

where $\dot{q}$, the local heat transfer rate, is

$$\dot{q} = \rho \ell^2 \frac{\partial h}{\partial y} \left|\frac{\partial u}{\partial y}\right| \tag{4.10}$$

and from our previous discussion of shear

$$\tau = \rho \ell^2 \frac{\partial u}{\partial y} \left|\frac{\partial u}{\partial y}\right| \tag{3.22}$$

Thus the total energy flux can be written

$$\dot{e} = \rho \ell^2 \left|\frac{\partial u}{\partial y}\right| \left[\frac{\partial h}{\partial y} + u \frac{\partial u}{\partial y}\right] = \rho \ell^2 \left|\frac{\partial u}{\partial y}\right| \frac{\partial h^0}{\partial y} \tag{4.11}$$

Now, utilizing our previous assumptions of a linear variation of stagnation enthalpy with velocity and a linear velocity profile

$$\frac{\partial h^0}{\partial y} = \frac{\partial h^0}{\partial u} \frac{\partial u}{\partial y} = \left(\frac{h_1^0 - h_2}{u_1}\right)\left(\frac{u_1}{\delta}\right) \tag{4.12}$$

and since from previously obtained results

$$\ell^2 \left|\frac{\partial u}{\partial y}\right| = \kappa \delta u_1$$

the equation for total energy transfer becomes

$$\dot{e} = \kappa \rho_* u_1 \left(h_1^0 - h_2\right) \qquad (4.13)$$

Here we have assumed, as was the case for the shear, that the characteristic value of a given transport should be computed on the dividing streamline.

There are several ways in which this quantity can be presented. Commonly the Stanton number is utilized. Here, from a comparison of the definition of Stanton number,

$$\dot{e} = \rho_1 u_1 St \left(h_1^0 - h_2\right) \qquad (4.14)$$

with (4.13), it can be seen that

$$St = \frac{\rho_*}{\rho_1} \kappa \qquad (4.15)$$

The density ratio on the dividing streamline may be computed by first calculating u_0/u_1 and η_*/a from (4.6) and (4.7), then u_*/u_1 from (3.3), and finally ρ_*/ρ_1 from (4.4). The shear stress proportionality constant κ is given in the previous section. The Stanton number thus computed for the range of Mach numbers and enthalpy ratios previously shown is given in Figure 7.

A second and perhaps here a more useful nondimensional parameter for the calculation of energy transfer may be

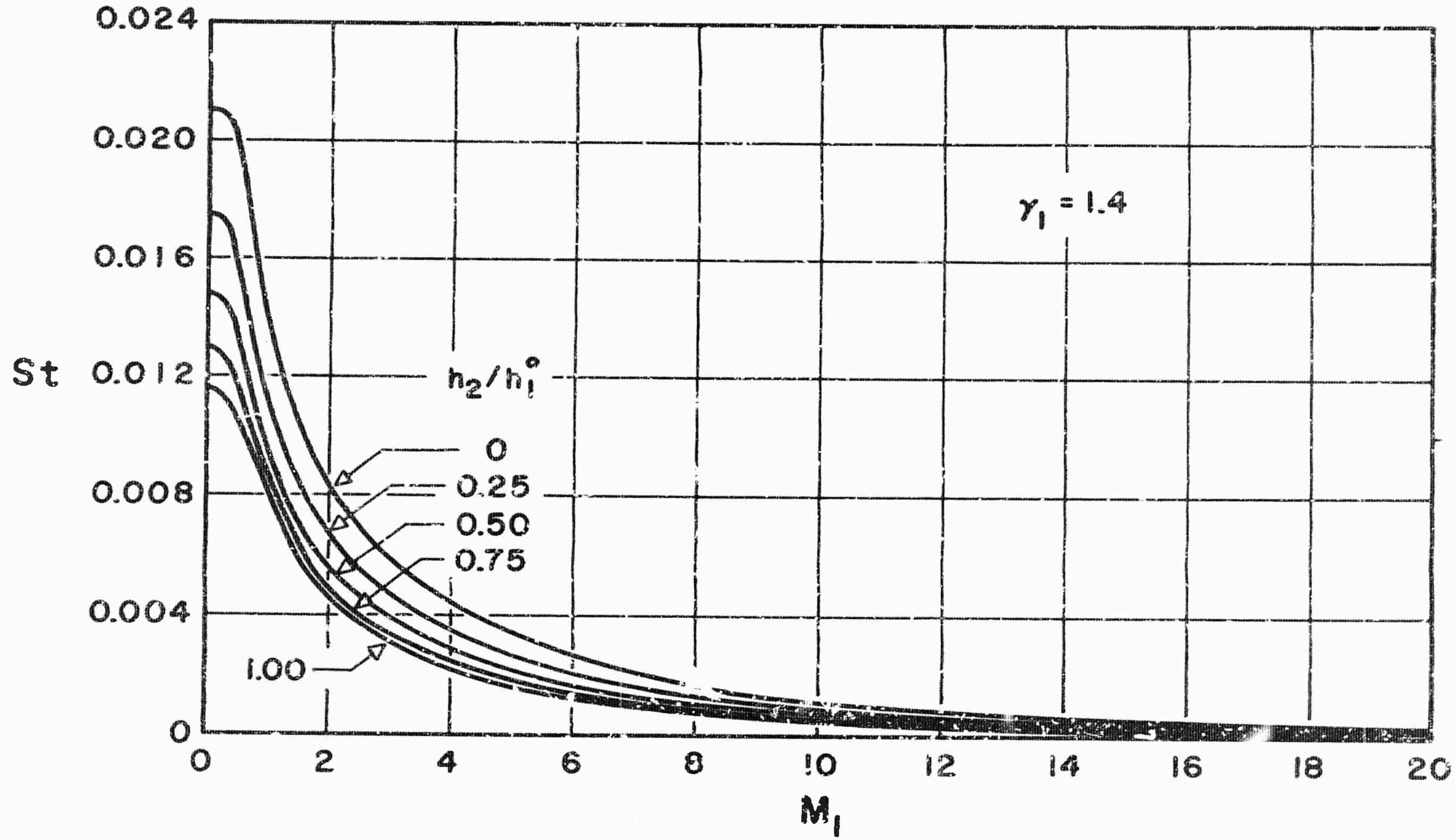

Figure 7. Variation of Stanton number - single gas.

obtained as follows. Expressing the density ρ_1 as (see (4.3))

$$\rho_1 = \frac{\gamma_1}{\gamma_1 - 1}\frac{p_1}{h_1} \tag{4.16}$$

the relation for energy transfer rate (4.14) can be expressed as

$$\dot{e} = \frac{\gamma_1}{\gamma_1 - 1} p_1 u_1 St \left[1 - \frac{h_2}{h_1^0}\right]\left(\frac{h_1^0}{h_1}\right) \tag{4.17}$$

Dividing (4.17) by $[\gamma_1/(\gamma_1 - 1)]p_1 u_1$, one obtains

$$\frac{\dot{e}}{\frac{\gamma_1}{\gamma_1 - 1} p_1 u_1} = St\left(1 - \frac{h_2}{h_1^0}\right)\left(\frac{h_1^0}{h_1}\right) \tag{4.18a}$$

or since $St = \frac{\rho_*}{\rho_1}\kappa$ there results finally

$$\frac{\dot{e}}{\frac{\gamma_1}{\gamma_1 - 1} p_1 u_1} = \frac{\rho_*}{\rho_1}\kappa\left(1 - \frac{h_2}{h_1^0}\right)\left(\frac{h_1^0}{h_1}\right) \tag{4.18b}$$

A plot of $\dot{e}/[\gamma_1/(\gamma_1 - 1)]p_1 u_1$ is given in Figure 8.

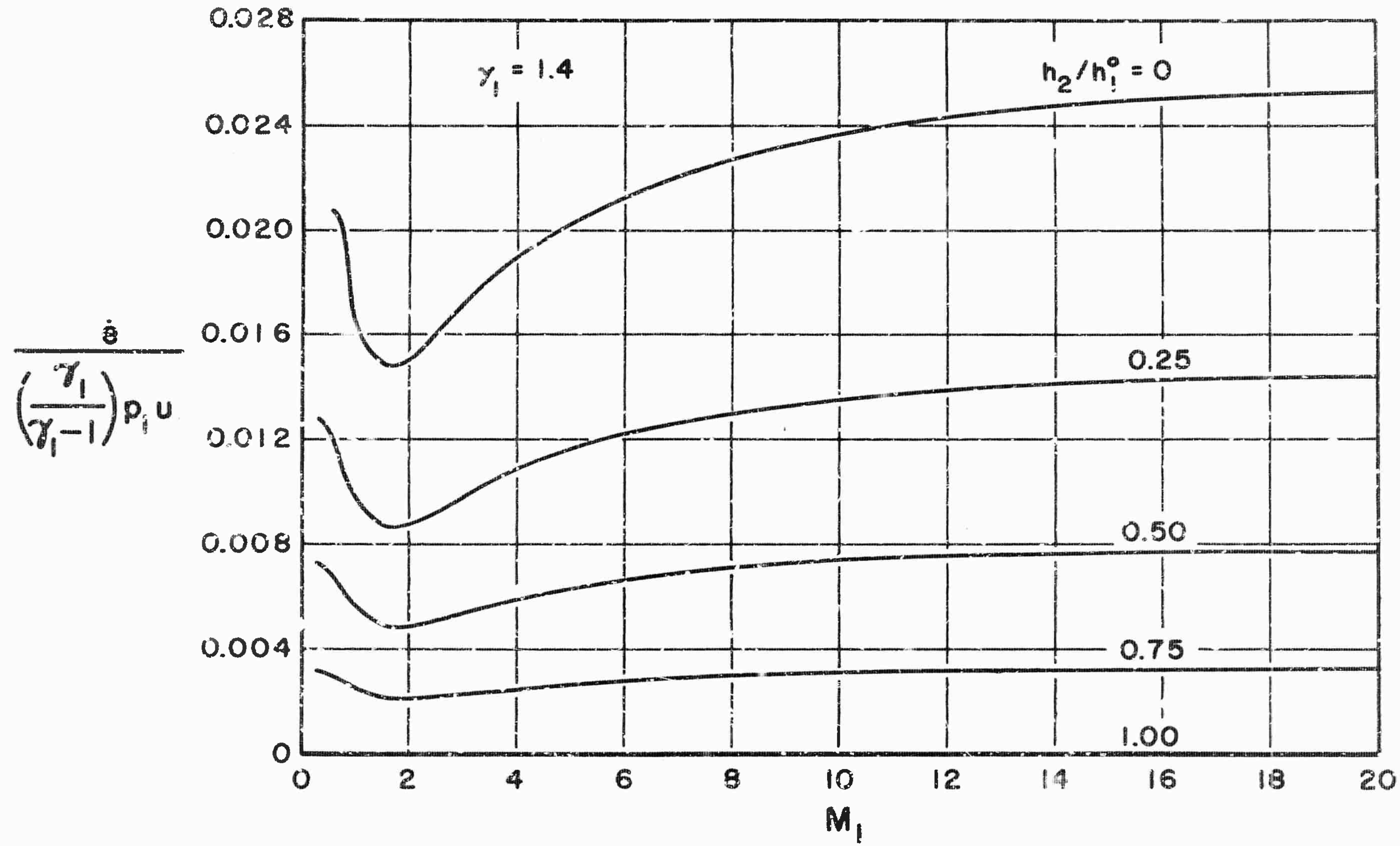

Figure 8. Variation of nondimensionalized energy transfer parameter - single gas.

5. COMPRESSIBLE TWO-GAS FLOW

In this section we will extend the analysis to include the mixing of two dissimilar gases.

5.1. Density distribution.

In what follows, we again utilize the development in reference 1. There it was shown that for the mixing of two species (in which no chemical reaction takes place) the local mass fraction of each species varies linearly across the mixing region. Letting c_α and c_β represent the mass fraction of the two species defined as

$$c_\alpha = \rho_\alpha/\rho \qquad \text{and} \qquad c_\beta = \rho_\beta/\rho \tag{5.1}$$

then from reference 1

$$c_\alpha = \frac{u}{u_1} \tag{5.2}$$

$$c_\beta = 1 - \frac{u}{u_1} \tag{5.3}$$

where α represents the species in the free stream.

With these relationships, we are in a position to reevaluate the local density expression. Assuming that the gases are still perfect

$$\rho = \frac{p}{RT} = \frac{p c_p m}{\mathcal{R} h} \tag{5.4}$$

where $\mathcal{R}$ is the universal gas constant and m the local molecular weight. Thus, for use in our mass flux and momentum flux integrals,

$$\frac{\rho}{\rho_1} = \frac{(c_p/c_{p1})(m/m_1)}{(h/h_1)} \tag{5.5}$$

from which we see that in addition to enthalpy ratio which is already available, we need expressions for the specific heat and molecular weight ratios.

The local enthalpy can be expressed as

$$c_p T = h = c_\alpha h_\alpha + c_\beta h_\beta = c_\alpha c_{p_1} T + c_\beta c_{p_2} T \tag{5.6}$$

if ideal gases are assumed as before. Eliminating the temperature

$$c_p = c_\alpha c_{p_1} + c_\beta c_{p_2} \tag{5.7}$$

or using (5.2) and (5.3)

$$c_p = \left(\frac{u}{u_1}\right) c_{p_1} + \left(1 - \frac{u}{u_1}\right) c_{p_2} \tag{5.8}$$

and so

$$\frac{c_p}{c_{p_1}} = \frac{u}{u_1}\left(1 - \frac{c_{p_2}}{c_{p_1}}\right) + \frac{c_{p_2}}{c_{p_1}} \tag{5.9}$$

The molecular weight can be expressed as

$$m = \frac{\rho}{n} \tag{5.10}$$

where n is the particle density. Noting that

$$c_\alpha \equiv \frac{\rho_\alpha}{\rho} = \frac{n_\alpha m_1}{\rho} \quad \text{and} \quad c_\beta \equiv \frac{\rho_\beta}{\rho} = \frac{n_\beta m_2}{\rho} \tag{5.11}$$

so that

$$n = n_\alpha + n_\beta = \left(\frac{c_\alpha}{m_1} + \frac{c_\beta}{m_2}\right) \tag{5.12}$$

we can write by combining (5.10) and (5.12)

$$m = \left[\frac{c_\alpha}{m_1} + \frac{c_\beta}{m_2}\right]^{-1} \tag{5.13}$$

Again, using (5.2) and (5.3)

$$m = \left[\frac{u/u_1}{m_1} + \frac{(1 - u/u_1)}{m_2}\right]^{-1} \tag{5.14}$$

or

$$\frac{m}{m_1} = \left[\frac{u}{u_1}\left(1 - \frac{m_1}{m_2}\right) + \frac{m_1}{m_2}\right]^{-1} \tag{5.15}$$

Returning to (5.5) and substituting for the specific heat, molecular weight, and enthalpy, expressions (5.9), (5.15), and (4.2b), we obtain the desired expression for the density.

$$\frac{\rho}{\rho_1} = \frac{\left[\frac{u}{u_1}\left(1 - \frac{c_{p_2}}{c_{p_1}}\right) + \frac{c_{p_2}}{c_{p_1}}\right]\left(\frac{h_1}{h_1^0}\right)}{\left[\frac{u}{u_1}\left(1 - \frac{m_1}{m_2}\right) + \frac{m_1}{m_2}\right]\left[\frac{h_2}{h_1^0} + \left(1 - \frac{h_2}{h_1^0}\right)\left(\frac{u}{u_1}\right) - \left(1 - \frac{h_1}{h_1^0}\right)\left(\frac{u}{u_1}\right)^2\right]} \tag{5.16}$$

5.2. Compressible two-gas flow parameters.

In section 4, it was demonstrated that once the proper density expression had been formulated in terms of the local velocity, the expression for all of the flow parameters of interest could be obtained in closed form. Having reevaluated the local density expression to account for the mixing of dissimilar gases, we may proceed in a manner identical to that outlined in section 4. The required integrations are considerably more involved but still tractable. The final expressions, now functions of c_{p_2}/c_{p_1} and m_1/m_2 as well as h_2/h_1^0, M_1, γ_1, and v_1/u_1 are given in the appendix. These also have been programmed for ARAP's digital computer. For maximum molecular weight effects, solutions were obtained for a ratio of entrained gas weight to free stream gas weight of both one-tenth and ten. In each case the entrained gas molecules were assumed to possess a large number of degrees of freedom so that a γ_2 of 1.2 was used throughout. To isolate the effects of the mixing of dissimilar gases, all computations assumed a stagnation enthalpy ratio of 0.5.

The same flow parameters as were presented for the single-gas compressible solutions are shown for two-gas flow in Figures 9 through 14. Note the very small variation of $a = \delta/x$ with molecular weight. The final plot, Figure 14, indicates that for the same external conditions, at high Mach numbers, the presence of a heavy entrained gas can be expected to decrease the energy transferred by approximately 65 percent relative to the case in which the entrained gas is a factor of 100 lighter.

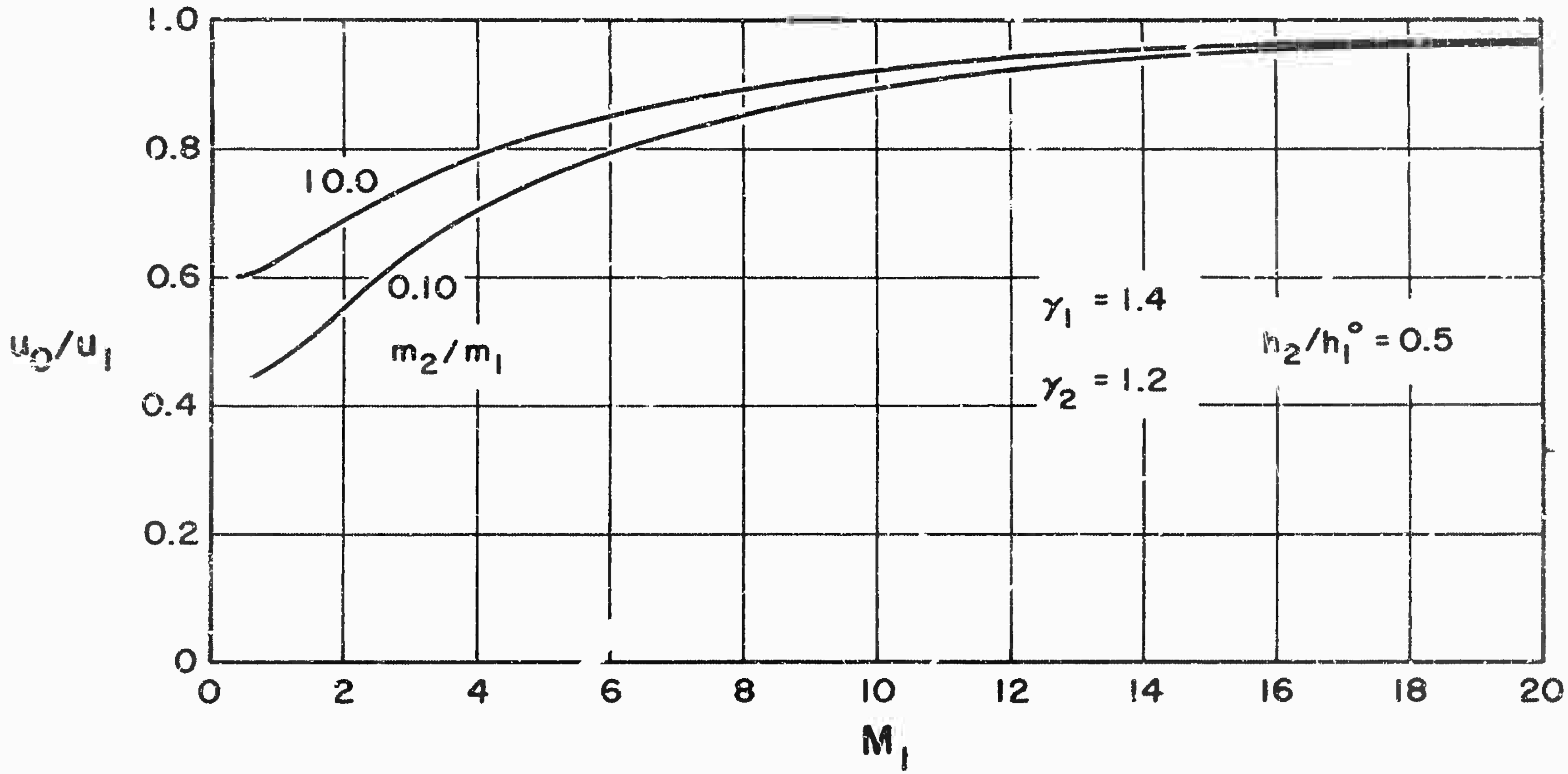

Figure 9. Variation of reference velocity - dissimilar gases.

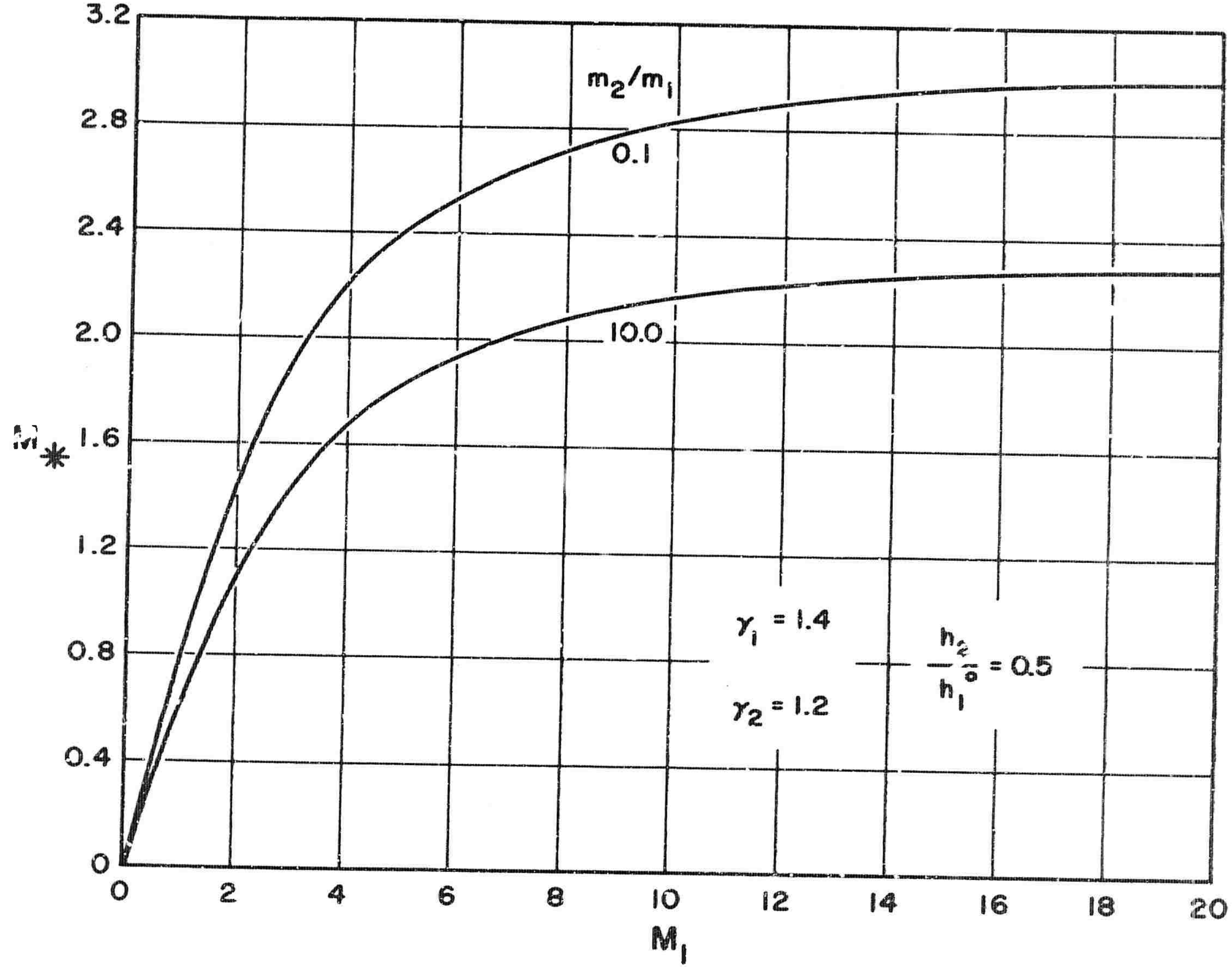

Figure 10. Varia 'on of local Mach number on the dividing streamline - dissimilar gases.

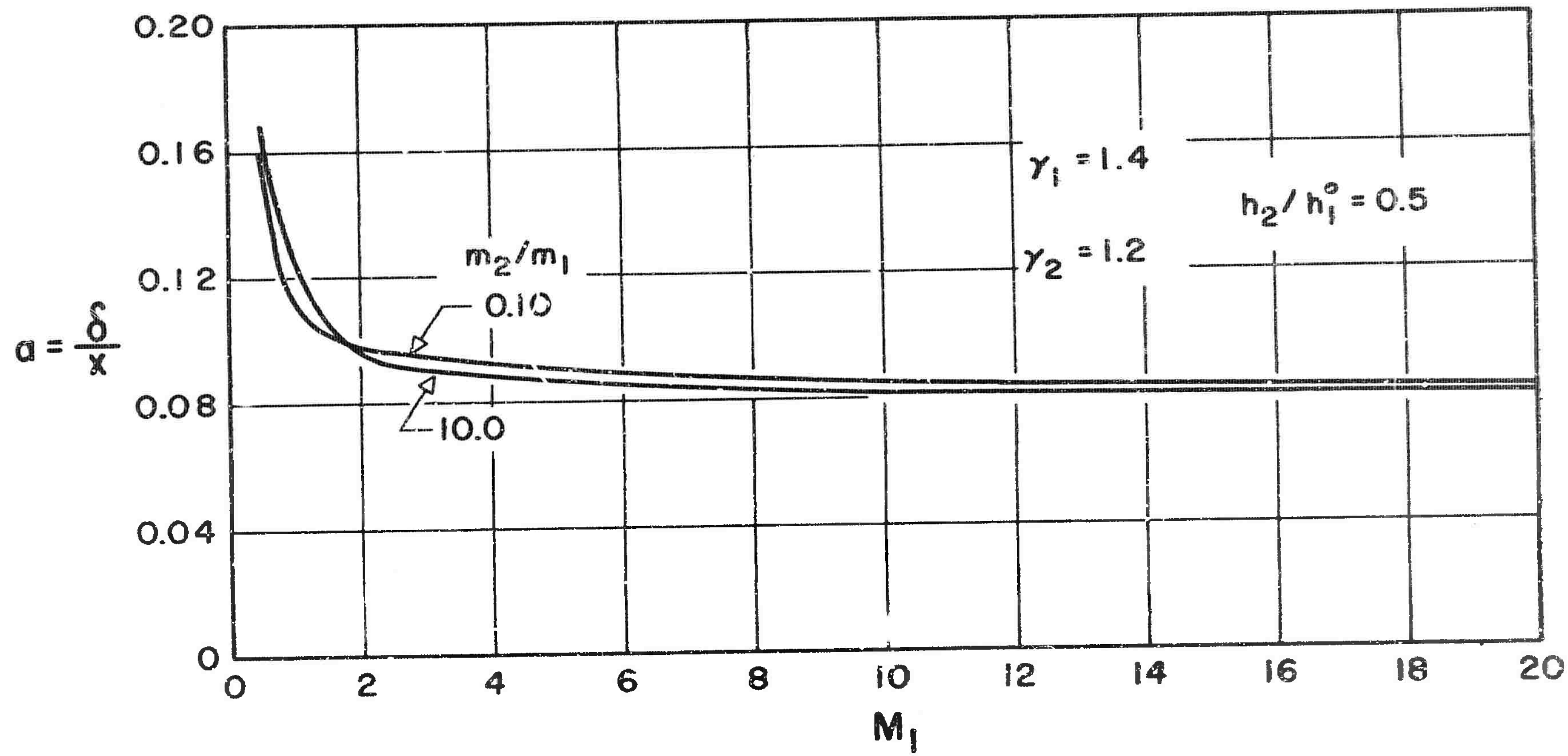

Figure 11. Variation of spreading rate parameter - dissimilar gases.

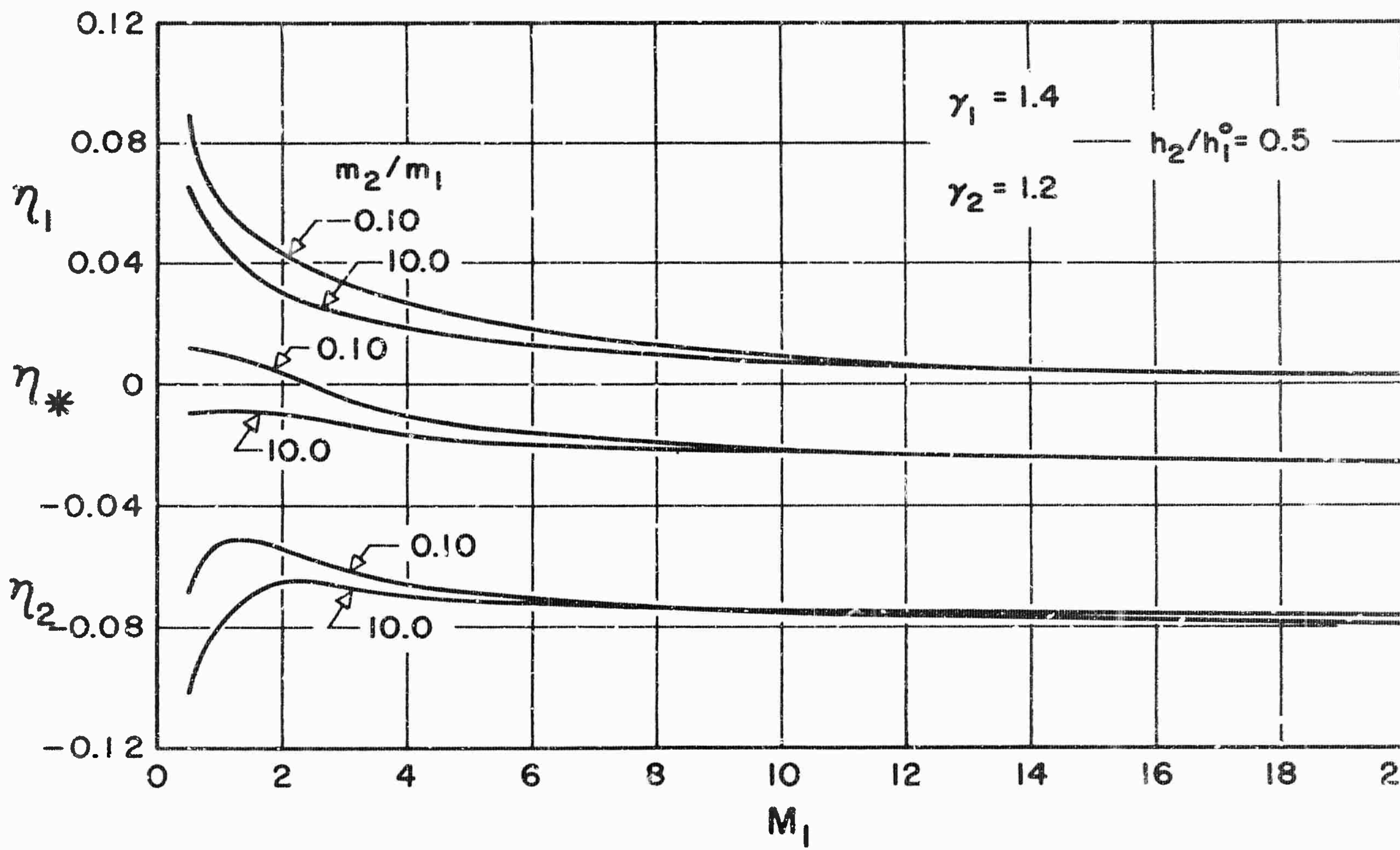

Figure 12. Variation of mixing region boundaries and dividing streamline - dissimilar gases.

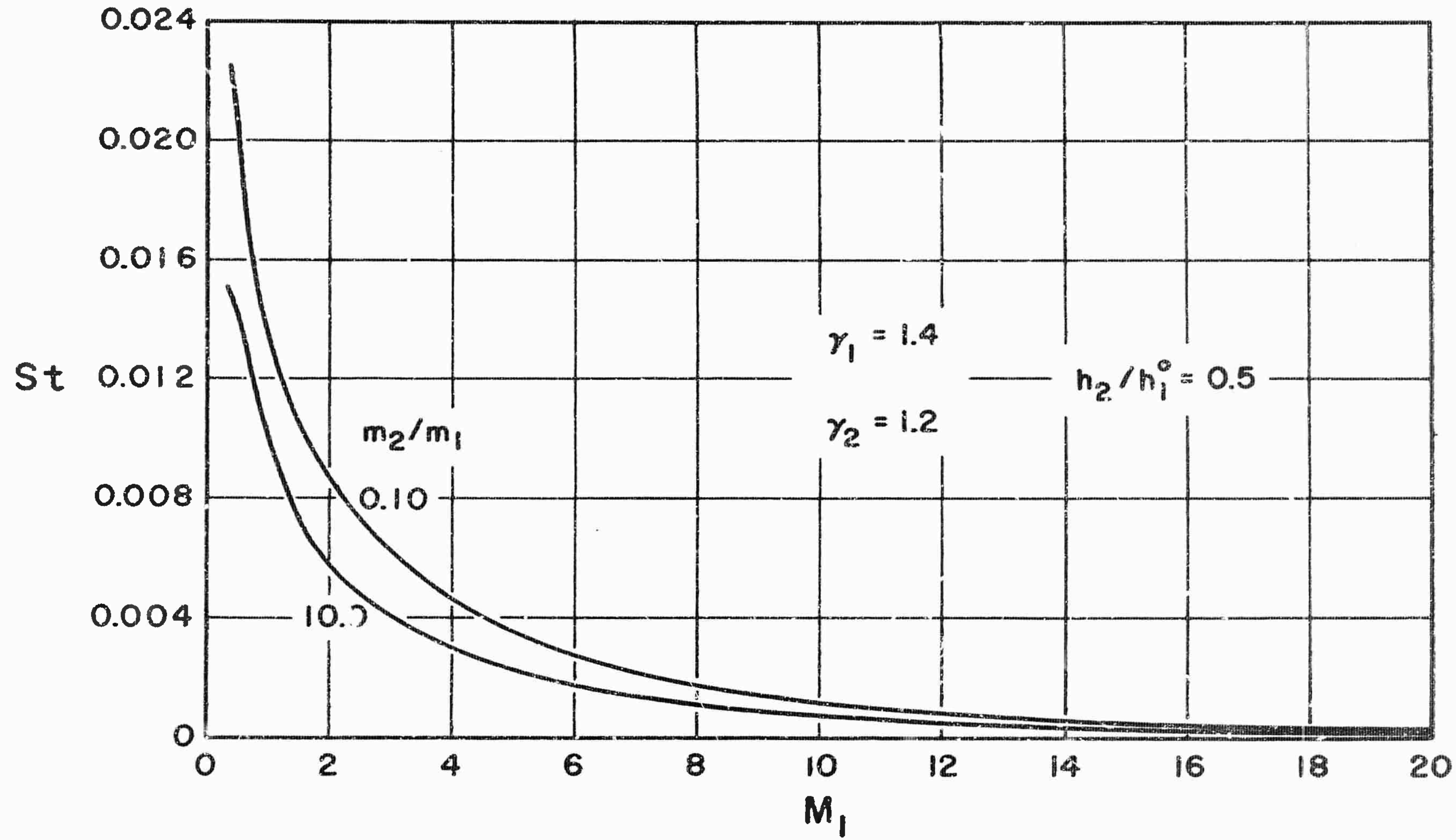

Figure 13. Variation of Stanton number - dissimilar gases.

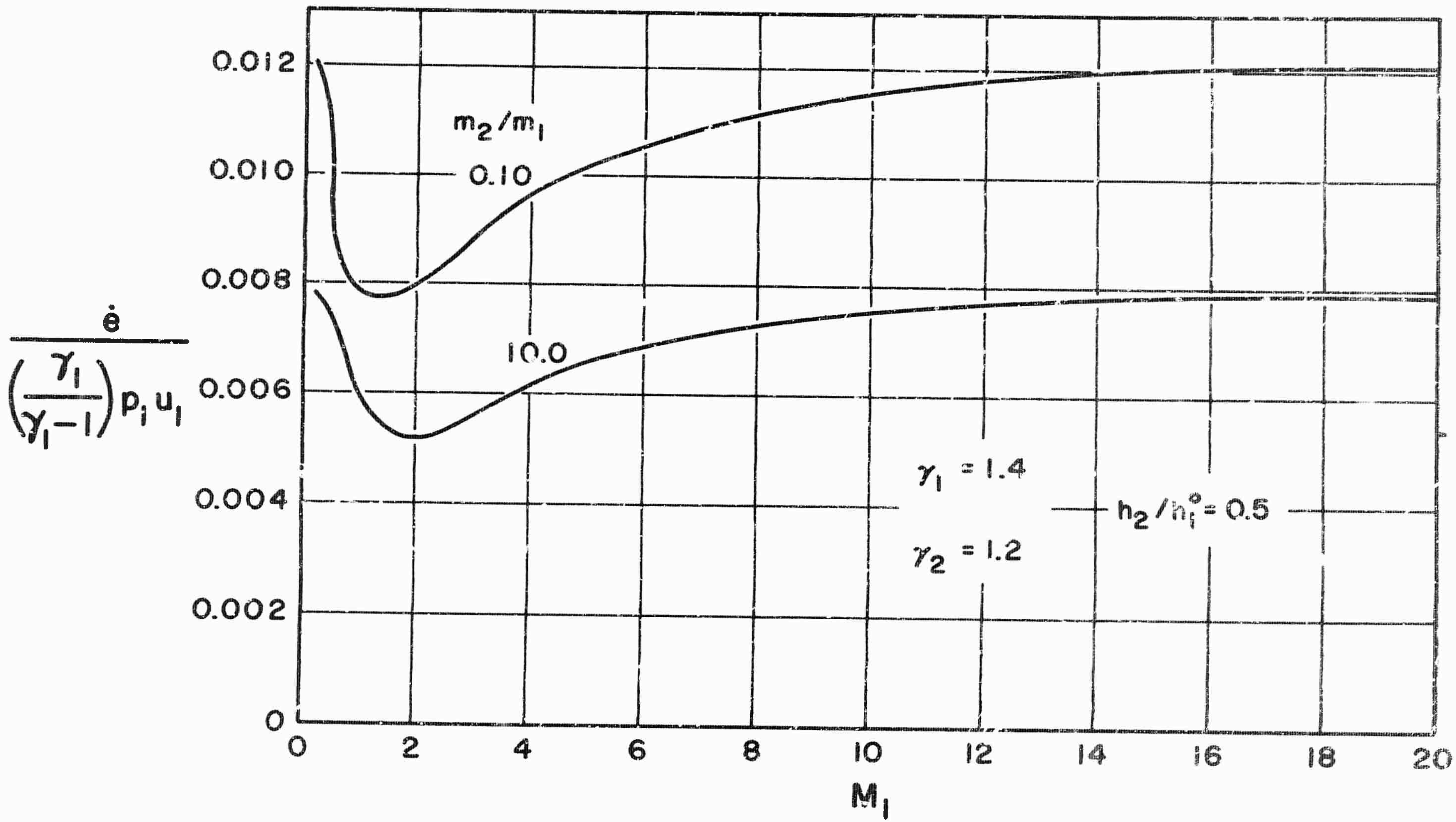

Figure 14. Variation of nondimensionalized energy transfer parameter - dissimilar gases.

6. CONCLUSIONS

By the use of several simplifying assumptions, closed form expressions have been obtained for the properties of a turbulent free shear layer in a compressible nonisoenergetic flow in which dissimilar gases are present on the two sides of the mixing region. Similar expressions are obtained for both a compressible and an incompressible single-gas shear layer.

Among the properties in the shear layer for which expressions have been developed are

a. the local velocities
b. the local thermodynamic quantities
c. the boundaries of the shear layer
d. the dividing streamline
e. the rate at which energy is transferred across the layer.

The specific equations are available in the appendix. Plots of these quantities as a function of Mach number, enthalpy ratio, and molecular weight are given in the body of this report.

7. REFERENCES

1. Donaldson, C.duP., and Gray, K.E. Theoretical and experimental investigation of the compressible free mixing of two dissimilar gases. ARAP Report No. 66, May 1965.

2. Tollmien, W. Berechnung turbulenter Ausbreitungs-vorgänge. ZAMM 6, 468-478 (1926).

APPENDIX

The relationships for the several properties of the single gas compressible shear layer are as follows:

Reference velocity.

$$\frac{u_0}{u_1} = f\left(\frac{h_2^0}{h_1^0}\,,\ M_1\,,\ \gamma_1\right)$$

$$\frac{u_0}{u_1} = 1 + \frac{2}{\psi} - \frac{2R}{\psi^2}\ln\phi - \frac{2}{\psi^2 k}\left(R^2 + \phi\psi\right)\ln\left(\frac{R + 2\phi + k}{R + 2\phi - \kappa}\right)$$

Dividing streamline.

$$g\left(\frac{\eta_*}{a}\,,\ \frac{h_2^0}{h_1^0}\,,\ M_1\,,\ \gamma_1\,,\ \frac{v_1}{u_1}\right) = 0$$

$$\left(\frac{B - \psi\left(\frac{\eta_*}{a}\right) + k}{B - \psi\left(\frac{\eta_*}{a}\right) - k}\right)\left(\frac{\rho_*}{\rho_1}\right)^{-\frac{k}{R}} = \left(\frac{R - \psi + k}{R - \psi - k}\right)\exp\left[\frac{\psi\kappa}{R}\left(1 - \frac{u_0}{u_1} - \frac{v_1}{au_1}\right)\right]$$

Spreading rate.

$$\frac{a}{\kappa} = \frac{\delta/x}{\kappa} = h\left(\frac{h_2^0}{h_1^0}\,,\ M_1\,,\ \gamma_1\,,\ \frac{\eta_*}{a}\right)$$

$$\frac{a}{\kappa} = \frac{\psi}{2}\left(\frac{\rho_*}{\rho_1}\right)\left[\left(\frac{R^2 + \psi\phi}{\psi k}\right)\ln\left(\frac{R - \psi\left(\frac{\eta_*}{a}\right) - \psi\frac{u_0}{u_1} - k}{R - \psi\left(\frac{\eta_*}{a}\right) - \psi\frac{u_0}{u_1} + k}\right) + \frac{R}{\psi}\ln\left(\frac{\rho_*}{\rho_1}\right)\right.$$

$$\left. - \frac{\eta_*}{a} - \frac{u_0}{u_1} + \left(\frac{R^2 + \psi\phi}{\psi k}\right)\ln\left(\frac{R + k}{R - k}\right) + \frac{R}{\psi}\ln\phi\right]^{-1}$$

where

$$\frac{\rho_*}{\rho_1} = \left[A + B\left(\frac{\eta_*}{a}\right) + C\left(\frac{\eta_*}{a}\right)^2\right]^{-1}$$

$$A = \phi + R\frac{u_0}{u_1} - \frac{\psi}{2}\left(\frac{u_0}{u_1}\right)^2$$

$$B = R - \psi\left(\frac{u_0}{u_1}\right)$$

$$C = -\frac{\psi}{2}$$

$$k = \left[2\psi\phi + R^2\right]^{1/2}$$

$$R = 1 + \frac{\psi}{2} - \phi$$

$$\phi = \frac{h_2}{h_1} = \frac{h_2}{h_1^0}\left(1 + \frac{\psi}{2}\right)$$

$$\psi = (\gamma_1 - 1)M_1^2$$

For the dissimilar gas compressible problem, these expressions are as follows:

Reference velocity.

$$\frac{u_0}{u_1} = f\left(\frac{h_2^0}{h_1^0},\ M_1,\ \gamma_1,\ \frac{m_2}{m_1},\ \frac{c_{p_2}}{c_{p_1}}\right)$$

$$\frac{u_0}{u_1} = 1 + \frac{2}{\psi}\left(\frac{1-P}{1-M}\right) - \left(\frac{1-P}{1-M}\right)\left[\frac{(\bar{P}-\bar{M})\,\bar{M}^2}{2(\phi - R\bar{M} - \frac{\psi}{2}\bar{M}^2)} + \frac{2(R+\psi\bar{M})}{\psi^2}\right.$$

$$\left. + \frac{\bar{P}-3\bar{M}}{\psi}\right] \ln\phi - \frac{1}{k}\left(\frac{1-P}{1-M}\right)\left[(3\bar{M}-2\bar{P})\bar{M} - \frac{(R+\psi\bar{M})(\bar{P}-\bar{M})\bar{M}^2}{2(\phi - R\bar{M} - \frac{\psi}{2}\bar{M}^2)}\right.$$

$$\left. + \frac{(R+\psi\bar{M})\,(\bar{P}-3\bar{M})}{\psi} + \frac{(R+\psi\bar{M})^2 + \psi(\phi - R\bar{M} - \frac{\psi}{2}\bar{M}^2)}{\psi^2/2}\right]$$

$$\ln\left|\frac{R+2\phi+k}{R+2\phi-k}\right| + \left(\frac{1-P}{1-M}\right)\frac{(\bar{P}-\bar{M})\bar{M}^2}{2(\phi - R\bar{M} - \frac{\psi}{2}\bar{M}^2)}\ \ln M$$

Dividing streamline.

$$g\left(\frac{\eta_*}{a}, \frac{h_2}{h_1^0}, M_1, \gamma_1, \frac{m_2}{m_1}, \frac{c_{p_2}}{c_{p_1}}\right) = 0$$

$$\left(1 - \frac{u_0}{u_1}\right)\left(\frac{1 - M}{1 - P}\right) = \frac{M(\bar{P} - \bar{M})}{(\phi - \frac{\psi}{2}\bar{M} - M)} \ln\left[\left(\frac{\eta_*}{a} + \frac{u_0}{u_1} + \bar{M}\right)(1 - M)\right]$$

$$+ \left[\frac{1}{\psi} - \frac{M(\bar{P} - \bar{M})}{2(\phi - \frac{\psi}{2}\bar{M} - M)}\right] \ln\left[A + B\frac{\eta_*}{a} + C\left(\frac{\eta_*}{a}\right)^2\right]$$

$$+ \frac{1}{k}\left[\bar{P} - 2\bar{M} + \frac{R + \psi\bar{M}}{\psi} + \frac{(R + \psi\bar{M})\bar{M}(\bar{P} - \bar{M})}{2[\phi(1 + \bar{M}) - \frac{\psi}{2}\bar{M}(1 + \bar{M}) - \bar{M}]}\right]$$

$$\ln\left(\frac{R - \psi - k}{R - \psi + k}\right)\left(\frac{B - \psi\frac{\eta_*}{a} + k}{B - \psi\frac{\eta_*}{a} - k}\right)$$

Spreading rate.

$$\frac{a}{\kappa} = \frac{\delta/x}{\kappa} = h\left(\frac{h_2}{h_1}\ ,\ M_1\ ,\ \gamma_1\ ,\ \frac{\eta_*}{a}\ ,\ \frac{m_2}{m_1}\ ,\ \frac{c_{p_2}}{c_{p_1}}\right)$$

$$\frac{a}{\kappa} = \left(\frac{\rho_*}{\rho_1}\right)\left(\frac{1 - M}{1 - P}\right)\left\{\left[\frac{(\bar{P} - \bar{M})\bar{M}^2}{(\phi - R\bar{M} - \frac{\psi}{2}\bar{M}^2)}\right]\ln\left(\frac{\frac{u_0}{u_1} + \bar{M} + \frac{\eta_*}{a}}{\bar{M}}\right)\right.$$

$$- \left[\frac{(\bar{P} - \bar{M})\bar{M}^2}{2(\phi - R\bar{M} - \frac{\psi}{2}\bar{M}^2)} + \frac{\bar{P} - 3\bar{M}}{\psi} + \frac{2(\bar{R} + \psi\bar{M})}{\psi^2}\right]\ln\left(\frac{A + B\frac{\eta_*}{a} + C\left(\frac{\eta_*}{a}\right)^2}{\phi}\right)$$

$$- \frac{2}{\psi}\left(\frac{\eta_*}{\varepsilon} + \frac{u_0}{u_1}\right)$$

$$+ \frac{1}{k}\left[\bar{M}(3\bar{M} - 2\bar{P}) - \frac{(R + \psi\bar{M})(\bar{P} - \bar{M})\bar{M}^2}{2(\phi - R\bar{M} - \frac{\psi}{2}\bar{M}^2)} + \frac{(R + \psi\bar{M})(\bar{P} - 3\bar{M})}{\psi}\right.$$

$$\left.\left. + \frac{(R + \psi\bar{M})^2 + \psi(\phi - R\bar{M} - \frac{\psi}{2}\bar{M}^2)}{\psi^2/2}\right]\ln\left(\frac{R + k}{R - k}\right)\left(\frac{2C\frac{\eta_*}{a} + B - k}{2C\frac{\eta_*}{a} + B + k}\right)\right\}^{-1}$$

where

$$\frac{\rho_*}{\rho_1} = \left(\frac{1 - P}{1 - M}\right) \frac{\left(\frac{u_0}{u_1} + \frac{\eta_*}{a} + \bar{P}\right)}{\left(\frac{u_0}{u_1} + \frac{\eta_*}{a} + \bar{M}\right)} \left[A + B\,\frac{\eta_*}{a} + C\left(\frac{\eta_*}{a}\right)^2\right]^{-1}$$

$$\bar{M} = \frac{M}{1 - M} \qquad M = \frac{m_1}{m_2}$$

$$\bar{P} = \frac{P}{1 - P} \qquad P = \frac{c_{p_2}}{c_{p_1}}$$

and R , A , B , C , k , ϕ , and ψ as above.

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13. ABSTRACT

An analytical study of the turbulent mixing of two dissimilar gases in a free shear layer was performed by ARAP for the Hypervelocity Kill Mechanisms Program. It was felt desirable to develop a simplfied method by which closed form expressions could be obtained for the characteristics of such a flow. These included the boundaries of the mixing region, the location of the dividing streamline, the local flow velocities, the local thermodynamic conditions, and the mean rate of energy transport across the shear layer.

It will be shown that although the effects of compressibility and the presence of dissimilar gases on either side of the mixing region raise the level of complexity of the problem, the approach remains the same as that used to solve the single gas compressible problem. Therefore, in order to focus attention on this approach, the incompressible one-gas flow is treated first (Section 3), then the modifications required to account for compressibility are shown and the results, which then become functions of Mach number and enthalpy, are given (Section 4), and finally, the treatment and results for the mixing of two dissimilar gases are presented (Section 5).

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